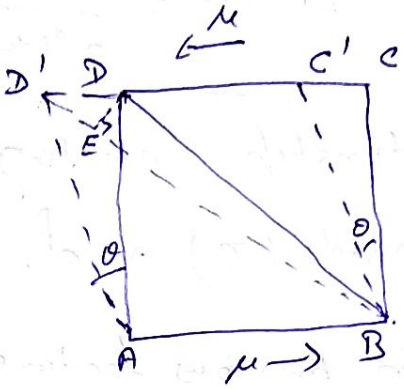
Relation connecting Young's modulus (Y), modulus of ~~rigidity~~ rigidity (η) and Poisson's ratio (σ).



In the last lecture note we have studied that if a square block ABCD of thickness unity is subjected ~~to shear stress~~ to a set of shear stresses of intensity μ on faces AB, CD and faces AD, CB, then the diagonal strain ~~along~~ is expressed ~~as~~ in terms of poisson ratio and μ ~~as~~ given by

$$\frac{\mu}{Y}(1+\sigma) \quad \text{——} (1)$$

$\therefore$ Total tensile strain along BD $= \frac{\mu}{Y}(1+\sigma)$ —— (2)

Since we have also seen that tensile strain in diagonal BD can be expressed in terms of Shear strain as $\frac{1}{2}$. shear Strain. —— (3)

From (2) and (3)

$$\frac{\mu}{Y}(1+\sigma) = \frac{1}{2} \times \text{shear strain}$$

$$= \frac{1}{2} \times \frac{\text{shear stress}}{\eta},$$

where, $\eta = \frac{\text{shear stress}}{\text{shear strain}}$ = modulus of rigidity

$$\Rightarrow \frac{\mu}{Y}(1+\sigma) = \frac{1}{2}\cdot\frac{\mu}{\eta}$$

or $$\frac{1+\sigma}{Y} = \frac{1}{2\eta}$$ ——

or $Y = 2\eta(1+\sigma)$ — (4)

or $\boxed{\eta = \frac{Y}{2(1+\sigma)}}$ — (5)

Relationship connecting Bulk modulus (K), modulus of rigidity (η) and Poisson's ratio (σ) :—

In previous lecture notes we have obtained the relations in ~~terms~~ Y, K and σ given by

$$Y = 3K(1-2\sigma) \quad \text{—} \quad (6)$$

Now equating equation (4) and (6)

$$2\eta(1+\sigma) = 3K(1-2\sigma)$$

$$\Rightarrow 3K - 2\eta = 6K\sigma + 2\eta\sigma$$

$$= \sigma(6K+2\eta)$$

or $\boxed{\sigma = \frac{3K-2\eta}{6K+2\eta}}$ — (7)